GEOMETRY

Junior High School

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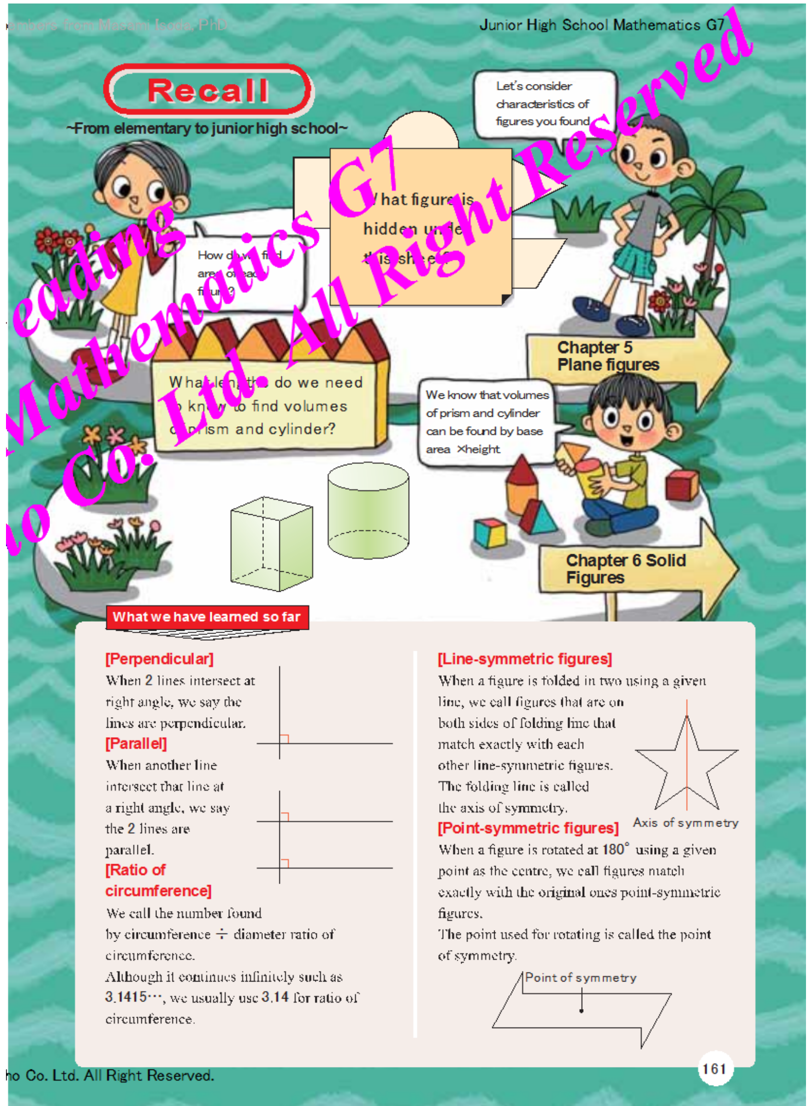
JH Grade 1

Features:

❑Includes Recall at the beginning of the chapter



JH Grade 1

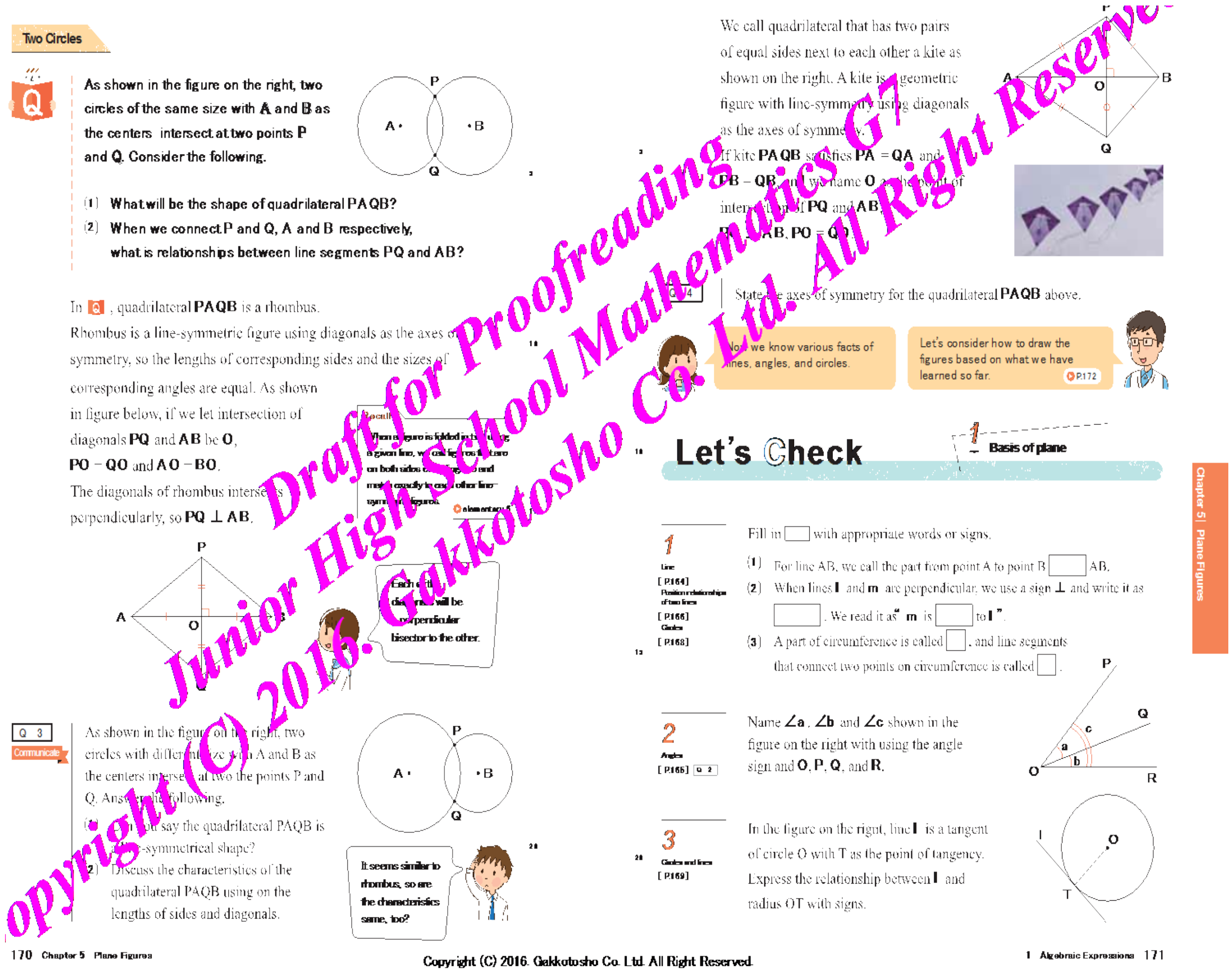
Features:

- ❑Uses real pictures, drawings, diagrams
- ❑Considers gender equality
- ❑Challenges students to think

JH Grade 1

Features:

- ❑Encourages students to explore and investigate
- ❑Assesses student learning after each lesson (Let's Check)
- ❑Mathematical connections
- ❑Recall is also found within the lesson


Two Circles

Q As shown in the figure on the right, two circles of the same size with A and B as the centers intersect at two points P and Q. Consider the following.

(1) What will be the shape of quadrilateral PAQB?

(2) When we connect P and Q, A and B respectively, what is relationships between line segments PQ and AB?

In Q, quadrilateral **PAQB** is a rhombus.
Rhombus is a line-symmetric figure using diagonals as the axes of symmetry, so the lengths of corresponding sides and the sizes of corresponding angles are equal. As shown in figure below, if we let intersection of diagonals **PQ** and **AB** be **O**,
PO = **QO** and **AO** = **BO**.
The diagonals of rhombus intersect perpendicularly, so **PQ** ⊥ **AB**.

Q3 Communicate As shown in the figure on the right, two circles with different size with A and B as the centers intersect at two the points P and Q. Answer the following.

(1) Can you say the quadrilateral PAQB is line-symmetrical shape?

(2) Discuss the characteristics of the quadrilateral PAQB using on the lengths of sides and diagonals.

We call quadrilateral that has two pairs of equal sides next to each other a kite as shown on the right. A kite is a geometric figure with line-symmetry using diagonals as the axes of symmetry.
If kite **PAQB** satisfies **PA** = **QA** and **PB** = **QB** and we name **O** as the point of intersection of **PQ** and **AB**, **PQ** ⊥ **AB**, **PO** = **QO**.

Q4 State the axes of symmetry for the quadrilateral **PAQB** above.

Now we know various facts of lines, angles, and circles.

Let's consider how to draw the figures based on what we have learned so far. P.172

Let's Check

1 Basis of plane

1 Fill in ☐ with appropriate words or signs.

(1) For line AB, we call the part from point A to point B ☐ AB.

(2) When lines **l** and **m** are perpendicular, we use a sign ⊥ and write it as ☐. We read it as " **m** is ☐ to **l** ".

(3) A part of circumference is called ☐, and line segments that connect two points on circumference is called ☐.

2 Name ∠**a**, ∠**b** and ∠**c** shown in the figure on the right with using the angle sign and **O**, **P**, **Q**, and **R**.

3 In the figure on the right, line **l** is a tangent of circle O with T as the point of tangency. Express the relationship between **l** and radius OT with signs.

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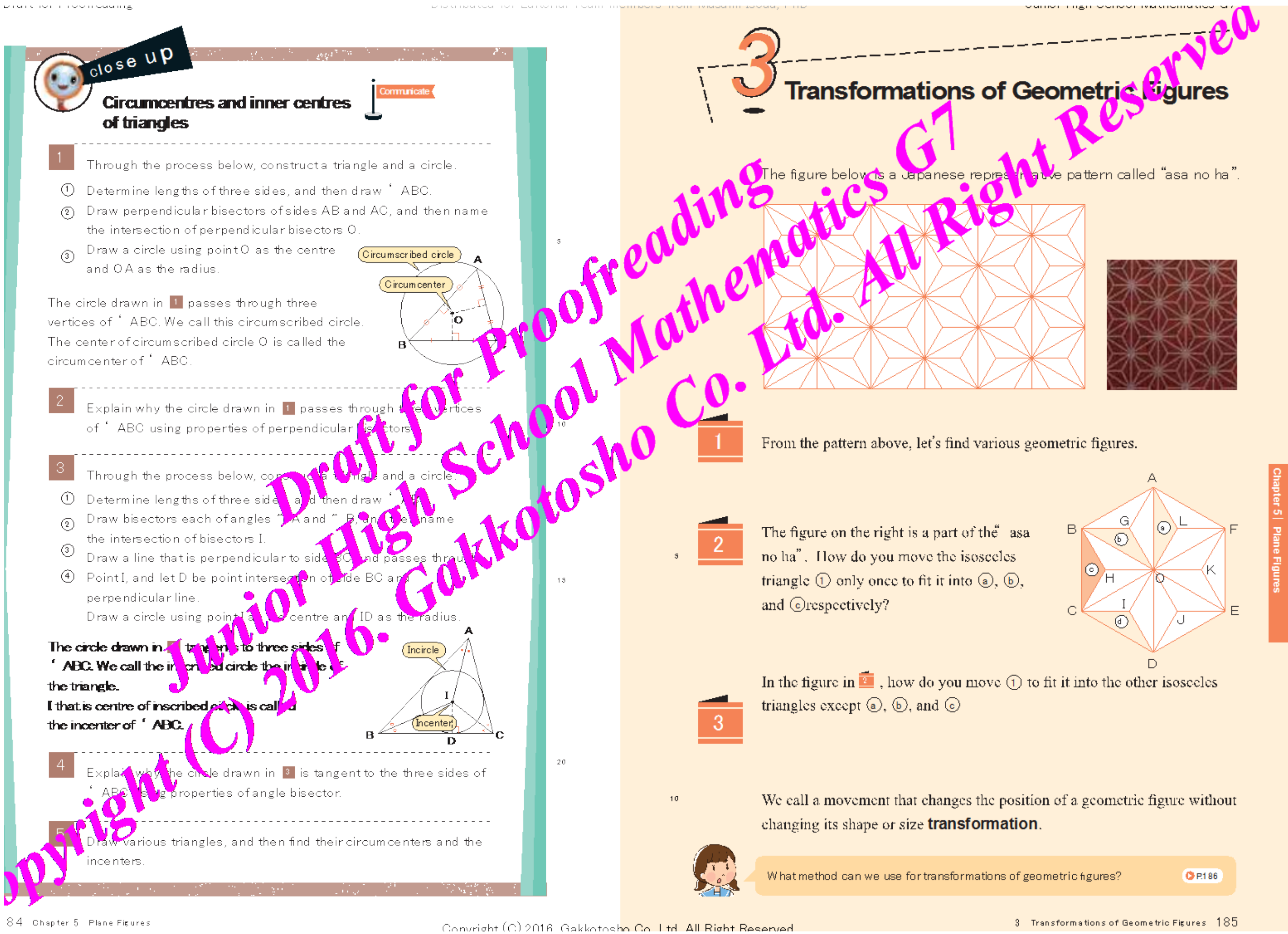
1 Algebraic Expressions 171

Chapter 5 | Plane Figures

JH Grade 1

Features:

- ❑Integrates Japanese culture
- ❑Further discussion of concepts (Close Up)
- ❑Emphasizes key words



close up

Circumcentres and inner centres of triangles

Communicate

1 Through the process below, construct a triangle and a circle.

① Determine lengths of three sides, and then draw ' ABC.

② Draw perpendicular bisectors of sides AB and AC, and then name the intersection of perpendicular bisectors O.

③ Draw a circle using point O as the centre and OA as the radius.

The circle drawn in 1 passes through three vertices of ' ABC. We call this circumscribed circle. The center of circumscribed circle O is called the circumcenter of ' ABC.

2 Explain why the circle drawn in 1 passes through the vertices of ' ABC using properties of perpendicular bisectors.

3 Through the process below, construct a triangle and a circle.

① Determine lengths of three sides, and then draw ' ABC.

② Draw bisectors each of angles ' A and ' B, and then name the intersection of bisectors I.

③ Draw a line that is perpendicular to side BC and passes through I.

④ Point I, and let D be point intersection of side BC and the perpendicular line.

Draw a circle using point I as the centre and ID as the radius.

The circle drawn in 3 is tangent to three sides of ' ABC. We call the inscribed circle the incircle of the triangle. I that is centre of inscribed circle is called the incenter of ' ABC.

4 Explain why the circle drawn in 3 is tangent to the three sides of ' ABC using properties of angle bisector.

5 Draw various triangles, and then find their circumcenters and the incenters.

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3 Transformations of Geometric Figures

The figure below is a Japanese representative pattern called "asa no ha".

1 From the pattern above, let's find various geometric figures.

2 The figure on the right is a part of the " asa no ha". How do you move the isosceles triangle ① only once to fit it into ⓐ, ⓑ, and ⓒ respectively?

3 In the figure in 2, how do you move ① to fit it into the other isosceles triangles except ⓐ, ⓑ, and ⓒ

We call a movement that changes the position of a geometric figure without changing its shape or size **transformation**.

What method can we use for transformations of geometric figures? P.186

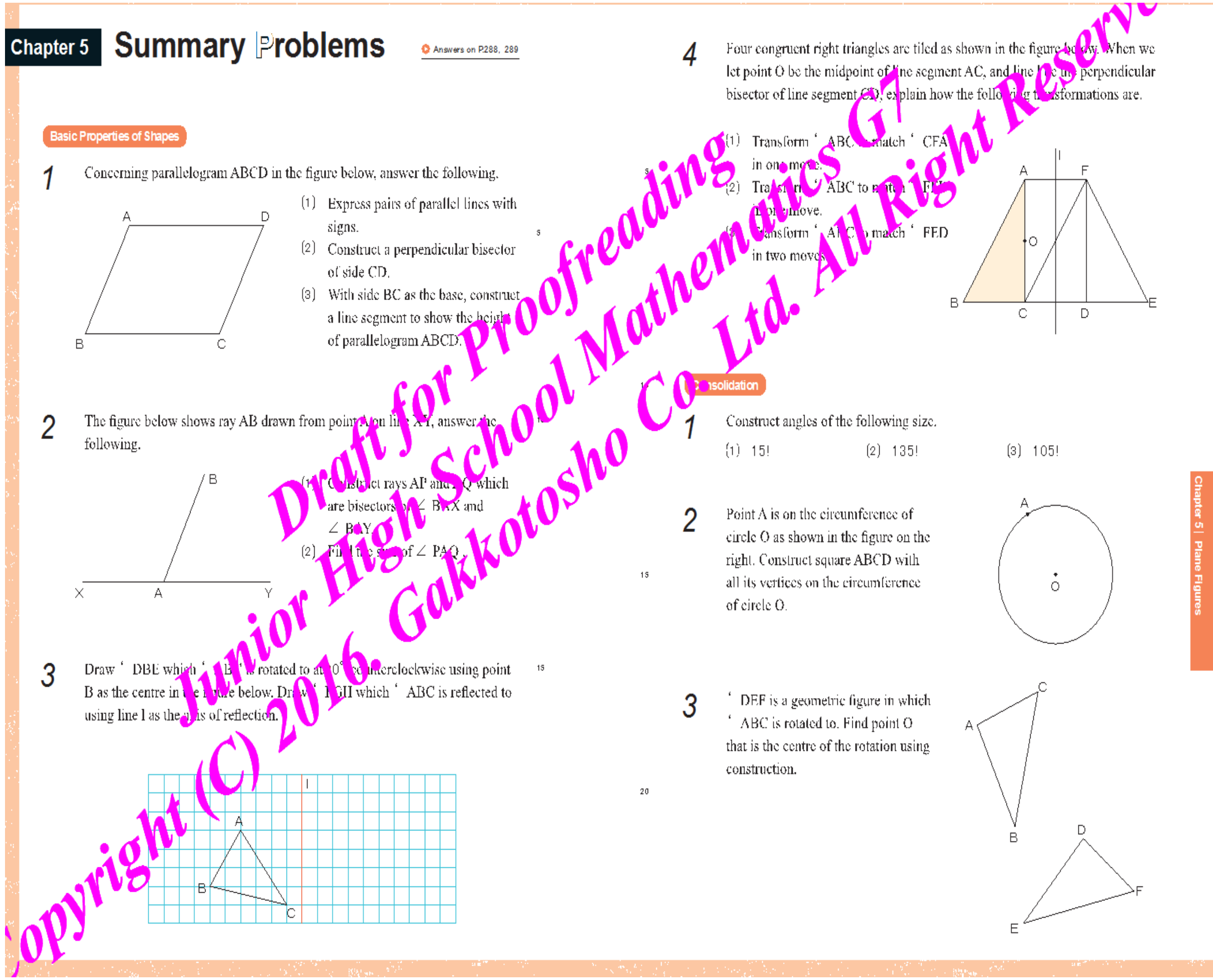
Chapter 5 | Plane Figures

3 Transformations of Geometric Figures 185

JH Grade 1

Features:

- ❑Assesses student learning of the lessons in the chapter
  (Summary Problems)



Chapter 5 **Summary Problems** Answers on P288, 289

Basic Properties of Shapes

1 Concerning parallelogram ABCD in the figure below, answer the following.

(1) Express pairs of parallel lines with signs.

(2) Construct a perpendicular bisector of side CD.

(3) With side BC as the base, construct a line segment to show the height of parallelogram ABCD.

2 The figure below shows ray AB drawn from point A on line XY, answer the following.

(1) Construct rays AP and AQ which are bisectors of ∠ BAX and ∠ BAY.

(2) Find the size of ∠ PAQ.

3 Draw ‘ DBE which ‘ ABC is rotated to a 90° counterclockwise using point B as the centre in the figure below. Draw ‘ GHI which ‘ ABC is reflected to using line l as the axis of reflection.

4 Four congruent right triangles are tiled as shown in the figure below. When we let point O be the midpoint of line segment AC, and line l be the perpendicular bisector of line segment CD, explain how the following transformations are.

(1) Transform ‘ ABC to match ‘ CFA in one move.

(2) Transform ‘ ABC to match ‘ FDC in one move.

(3) Transform ‘ ABC to match ‘ FED in two moves.

Consolidation

1 Construct angles of the following size.

(1) 15! (2) 135! (3) 105!

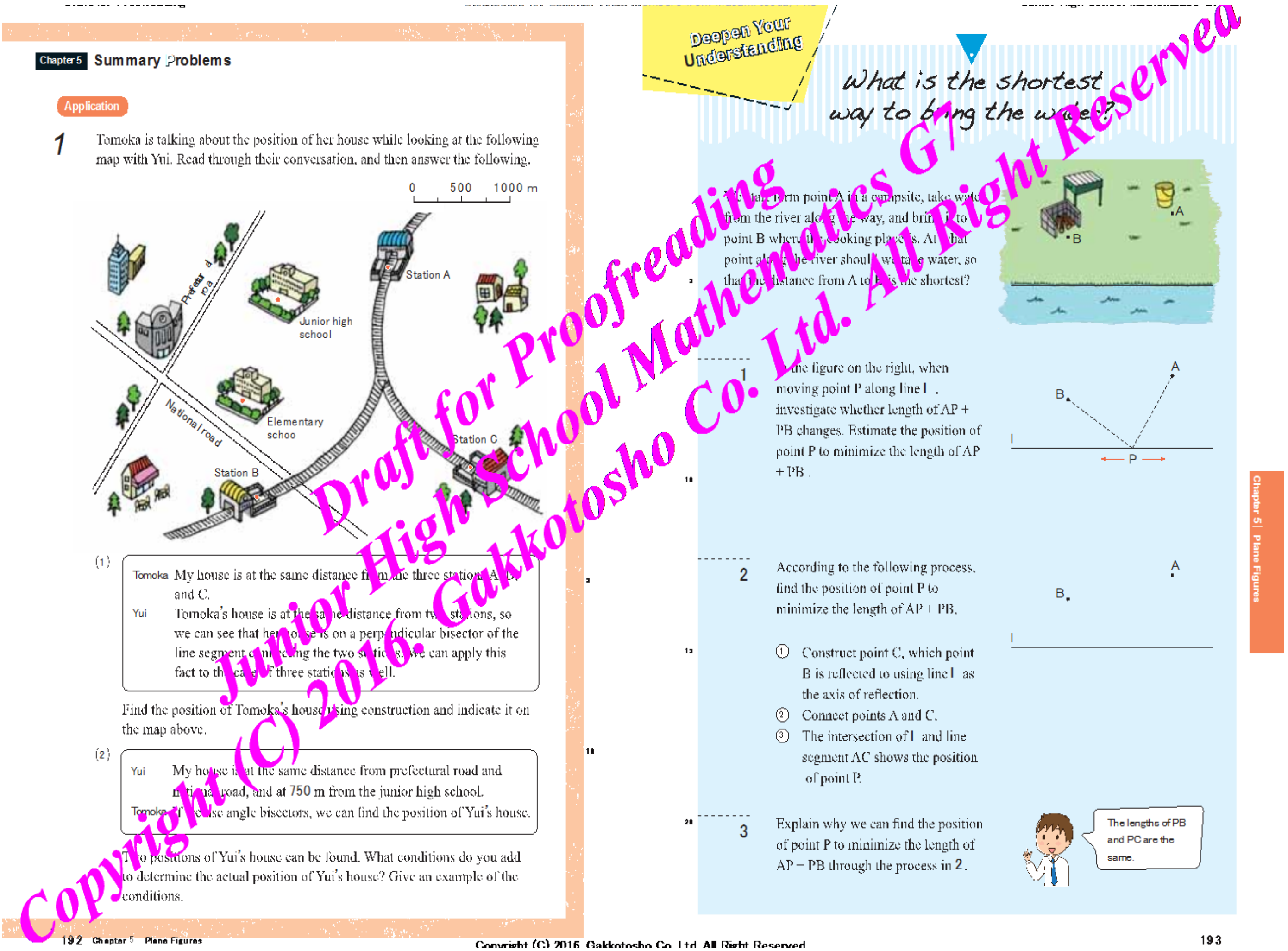
2 Point A is on the circumference of circle O as shown in the figure on the right. Construct square ABCD with all its vertices on the circumference of circle O.

3 ‘ DEF is a geometric figure in which ‘ ABC is rotated to. Find point O that is the centre of the rotation using construction.

JH Grade 1

Features:

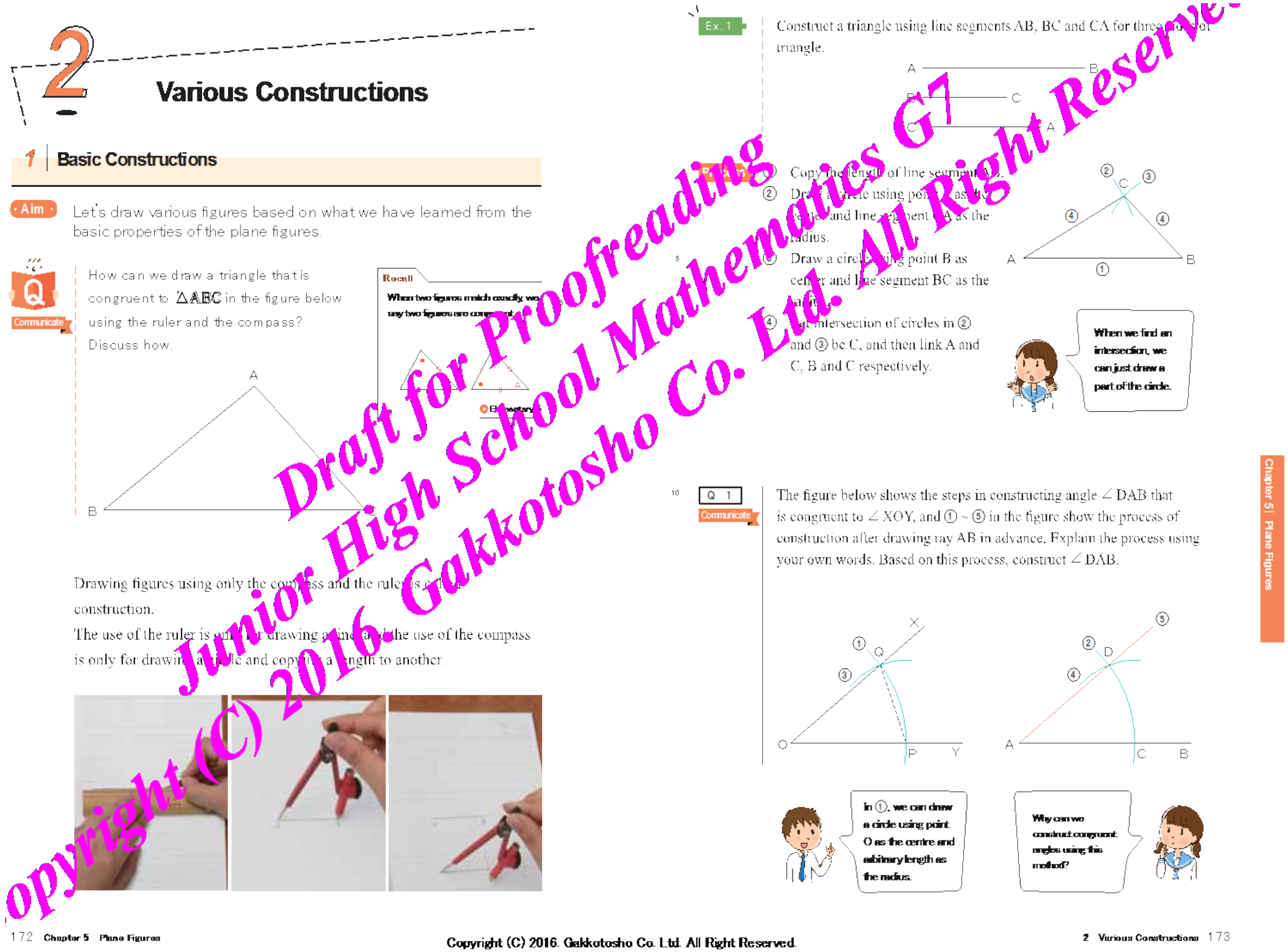
- ❑Deepens students' understanding through application to real life situations
  - (Deepen your Understanding)



JH Grade 1

Features:

- ❑Includes real pictures of students doing the task
- ❑Develops students' communication skills


2 Various Constructions

1 Basic Constructions

Aim Let's draw various figures based on what we have learned from the basic properties of the plane figures.

Q Communicate: How can we draw a triangle that is congruent to △ABC in the figure below using the ruler and the compass? Discuss how.

Recall: When two figures match exactly, we say two figures are congruent.

Drawing figures using only the compass and the ruler is called construction.

The use of the ruler is only for drawing a line and the use of the compass is only for drawing a circle and copying a length to another.

Ex. 1 Construct a triangle using line segments AB, BC and CA for three sides of triangle.

① Copy the length of line segment AB.
② Draw a circle using point A as the center and line segment CA as the radius.
③ Draw a circle using point B as center and line segment BC as the radius.
④ Let intersection of circles in ② and ③ be C, and then link A and C, B and C respectively.

When we find an intersection, we can just draw a part of the circle.

Q 1 Communicate: The figure below shows the steps in constructing angle ∠DAB that is congruent to ∠XOY, and ① ~ ⑤ in the figure show the process of construction after drawing ray AB in advance. Explain the process using your own words. Based on this process, construct ∠DAB.

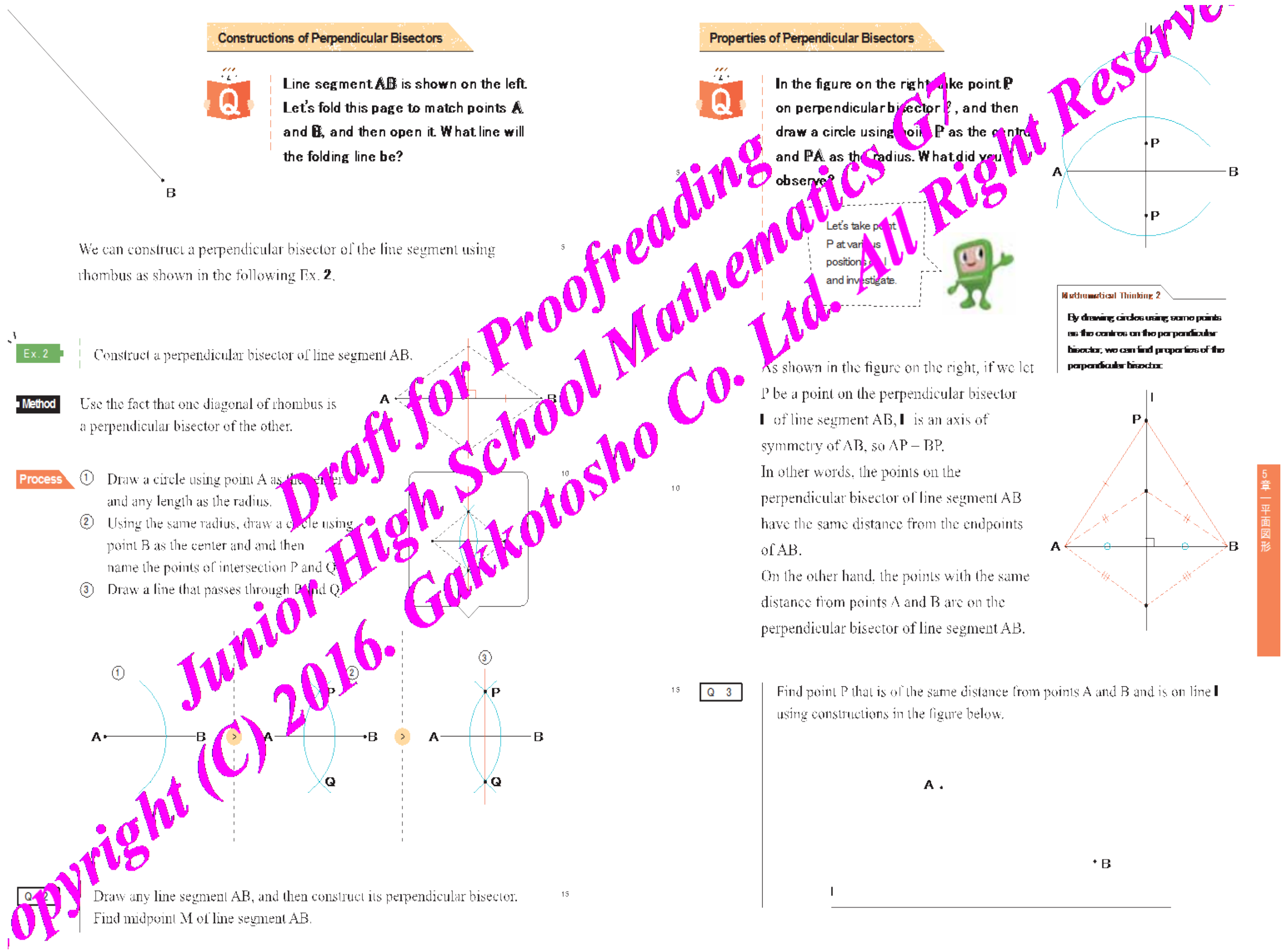
In ①, we can draw a circle using point O as the centre and arbitrary length as the radius.

Why can we construct congruent angles using this method?

JH Grade 1

Features:

❑Emphasizes mathematical thinking and investigation



Constructions of Perpendicular Bisectors

Q Line segment AB is shown on the left. Let's fold this page to match points A and B, and then open it. What line will the folding line be?

We can construct a perpendicular bisector of the line segment using rhombus as shown in the following Ex. 2.

Ex. 2 Construct a perpendicular bisector of line segment AB.

Method Use the fact that one diagonal of rhombus is a perpendicular bisector of the other.

Process
① Draw a circle using point A as the center and any length as the radius.
② Using the same radius, draw a circle using point B as the center and and then name the points of intersection P and Q.
③ Draw a line that passes through P and Q.

Q Draw any line segment AB, and then construct its perpendicular bisector. Find midpoint M of line segment AB.

Properties of Perpendicular Bisectors

Q In the figure on the right, take point P on perpendicular bisector ℓ, and then draw a circle using point P as the centre and PA as the radius. What did you observe?

Let's take point P at various positions and investigate.

Mathematical Thinking 2
By drawing circles using some points as the centres on the perpendicular bisector, we can find properties of the perpendicular bisector.

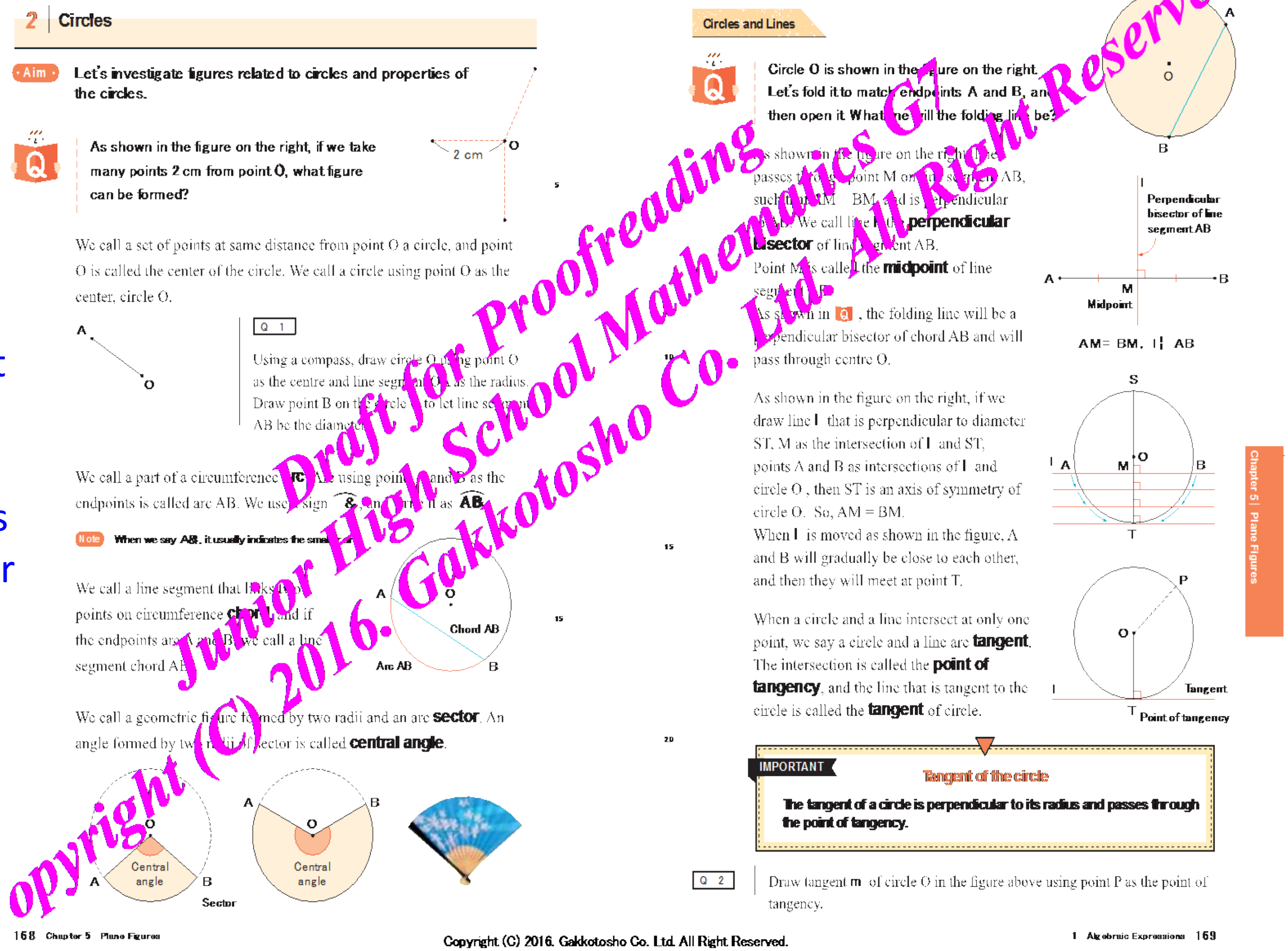
As shown in the figure on the right, if we let P be a point on the perpendicular bisector ℓ of line segment AB, ℓ is an axis of symmetry of AB, so AP = BP.
In other words, the points on the perpendicular bisector of line segment AB have the same distance from the endpoints of AB.
On the other hand, the points with the same distance from points A and B are on the perpendicular bisector of line segment AB.

Q 3 Find point P that is of the same distance from points A and B and is on line ℓ using constructions in the figure below.

JH Grade 1

Features:

- ❑Encourages students to think and investigate
- ❑Relates concept to real objects
- ❑Important ideas are emphasized or placed in a box



2 Circles

Aim Let's investigate figures related to circles and properties of the circles.

Q As shown in the figure on the right, if we take many points 2 cm from point O, what figure can be formed?

We call a set of points at same distance from point O a circle, and point O is called the center of the circle. We call a circle using point O as the center, circle O.

Q 1 Using a compass, draw circle O using point O as the centre and line segment OA as the radius. Draw point B on the circle to let line segment AB be the diameter.

We call a part of a circumference arc AB using points A and B as the endpoints is called arc AB. We use sign ⌒ and write it as $\overset{\frown}{AB}$.

Note When we say $\overset{\frown}{AB}$, it usually indicates the sma...

We call a line segment that links two points on circumference chord, and if the endpoints are A and B we call a line segment chord AB.

We call a geometric figure formed by two radii and an arc **sector**. An angle formed by two radii of sector is called **central angle**.

Circles and Lines

Q Circle O is shown in the figure on the right. Let's fold it to match endpoints A and B, and then open it. What line will the folding line be?

As shown in the figure on the right, line l passes through point M on line segment AB, such that AM = BM, and is perpendicular to AB. We call line l the **perpendicular bisector** of line segment AB. Point M is called the **midpoint** of line segment AB.

As shown in Q, the folding line will be a perpendicular bisector of chord AB and will pass through centre O.

As shown in the figure on the right, if we draw line l that is perpendicular to diameter ST, M as the intersection of l and ST, points A and B as intersections of l and circle O, then ST is an axis of symmetry of circle O. So, AM = BM.

When l is moved as shown in the figure, A and B will be gradually be close to each other, and then they will meet at point T.

When a circle and a line intersect at only one point, we say a circle and a line are **tangent**. The intersection is called the **point of tangency**, and the line that is tangent to the circle is called the **tangent** of circle.

> **IMPORTANT** Tangent of the circle
>
> The tangent of a circle is perpendicular to its radius and passes through the point of tangency.

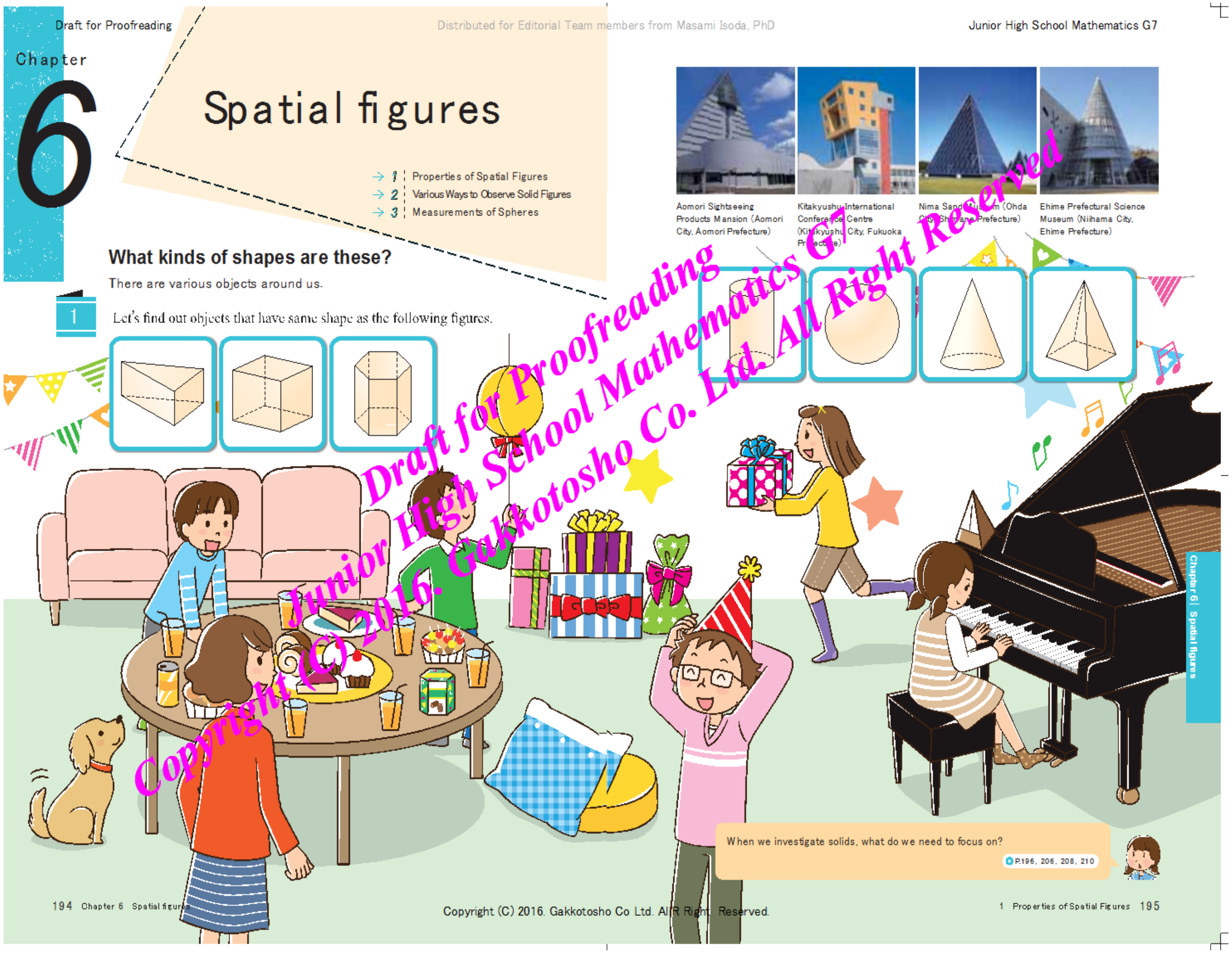
Q 2 Draw tangent m of circle O in the figure above using point P as the point of tangency.

JH Grade 1

Features:

- ❑Uses situations which are within the experience of the students

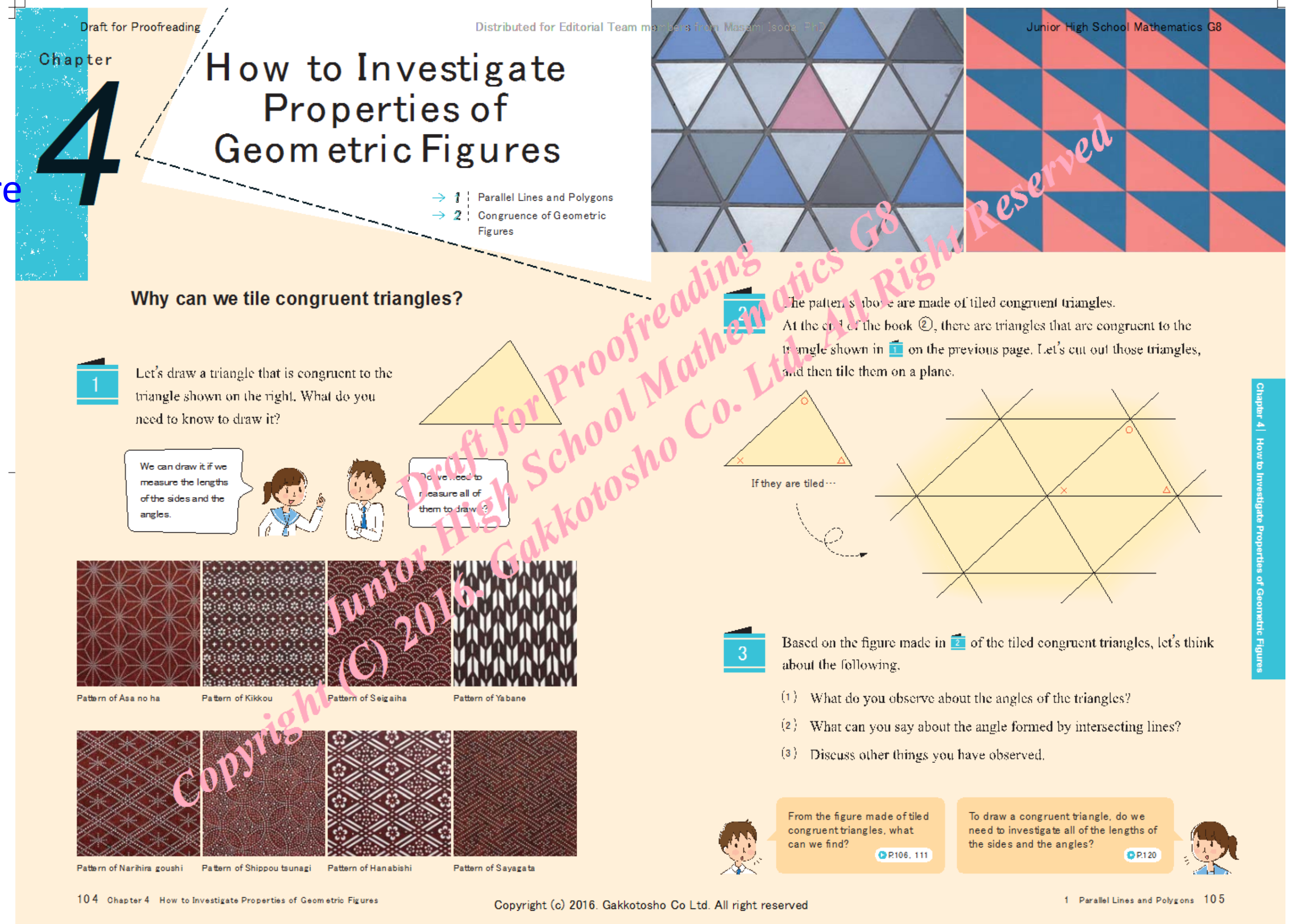
- ❑Relates ideas to real life structures or buildings found in different places in Japan



JH Grade 2

Features:

- ❑Promotes Japanese culture through the given task



Chapter 4

How to Investigate Properties of Geometric Figures

Why can we tile congruent triangles?

1 Let's draw a triangle that is congruent to the triangle shown on the right. What do you need to know to draw it?

Pattern of Asa no ha
Pattern of Kikkou
Pattern of Seigaiha
Pattern of Yabane
Pattern of Narihira goushi
Pattern of Shippou tsunagi
Pattern of Hanabishi
Pattern of Sayagata

2 The patterns above are made of tiled congruent triangles. At the end of the book ②, there are triangles that are congruent to the triangle shown in 1 on the previous page. Let's cut out those triangles, and then tile them on a plane.

3 Based on the figure made in 2 of the tiled congruent triangles, let's think about the following.

(1) What do you observe about the angles of the triangles?

(2) What can you say about the angle formed by intersecting lines?

(3) Discuss other things you have observed.

JH Grade 2

- Relates mathematical ideas with real life structures or buildings found not only in Japan but in other countries as well

Chapter 5

Triangles and Quadrilaterals

We can make various triangles and quadrilaterals using origami.

Can we make equilateral triangles and parallelograms by folding?

1 Let's look for various shapes of objects around us.

Shirakawa-go Gassho Style (Ono-gun, Gifu Prefecture)

(State of Colorado, USA)

Tokyo Budokan (Adachi Ward, Tokyo Metropolis)

Let's consider how to fold, and then make the following geometric figures.

Isosceles triangle

Remember how to construct each geometric figure.

Equilateral triangle

Parallelogram

2 Why can isosceles triangle, equilateral triangle, and parallelogram be made by folding in the way shown above? Explain why.

What are the properties of triangles and quadrilaterals? P.140, 151

Thank you for your comments and suggestions on the four chapters on Geometry.